NOTES

Electronic Calculators

1. The use of electronic calculators is prohibited in O level Mathematics, Paper 1 (4017/1), N level Mathematics Syllabus A, Paper 1 (4010/1) and N level Mathematics Syllabus T, Paper 1 (4012/1).
2. The use of silent electronic calculators is **expected** in O level Additional Mathematics (4018), O level Mathematics, Paper 2 (4017/2), N level Mathematics Syllabus A, Paper 2 (4010/2) and N level Mathematics Syllabus T, Paper 2 (4012/2).
3. More detailed regulations concerning the use of electronic calculators will be issued by the Examinations Branch of the Ministry of Education.

Lists of Formulae, etc.

Formulae for O level Additional Mathematics are printed on the question papers.

Mathematical Instruments

Apart from the usual mathematical instruments, candidates may use flexicurves in all the examinations.

Mathematical Notation

Attention is drawn to the list of mathematical notation on pages 8-11.

4017 MATHEMATICS O LEVEL

GCE O LEVEL
MATHEMATICS (4017)

LEARNING AIMS

The course aims to enable students to:

1. develop mathematical language as a means of communication;
2. acquire a foundation appropriate to a further study of Mathematics and skills and knowledge pertinent to other disciplines;
3. acquire and apply skills and knowledge relating to number, measure and space in mathematical situations that they will meet in life;
4. develop an understanding of mathematical principles and the abilities to reason logically;
5. conduct individual and co-operative enquiry and experiment, including extended pieces of work of a practical and investigative kind;
6. integrate information technology to enhance the mathematical experience;
7. engage in imaginative and creative work arising from mathematical ideas;
8. enhance their intellectual curiosity and appreciate the power and structure of Mathematics including patterns and relationships;
9. develop a positive attitude towards mathematics, including confidence, enjoyment and perseverance;
10. appreciate the interdependence between different branches of Mathematics.

ASSESSMENT OBJECTIVES

The examination will test the ability of candidates to:

1. recognise the appropriate mathematical procedures for a given situation;
2. perform calculations by suitable methods, with and without a calculating aid;
3. apply systems of measurement in everyday use and in the solutions of problems;
4. estimate, approximate and use appropriate degrees of accuracy;
5. interpret, use and present information in written, graphical, diagrammatic and tabular forms;
6. use geometrical instruments;
7. recognise and apply spatial relationships in two and three dimensions;
8. recognise patterns and structures in a variety of situations and form, and justify generalisations;
9. understand and use mathematical language and symbols effectively and present mathematical arguments and solutions to problems in a logical and clear fashion;
10. apply and interpret Mathematics in familiar and unfamiliar contexts, including daily life;
11. analyse problems and formulate them into mathematical terms; select, apply and communicate appropriate strategies and techniques to obtain the solutions; check the results; and interpret the solutions in terms of the problems.

EXAMINATION

Scheme of Papers

Component	*Time Allocation*	*Type*	*Maximum Mark*	*Weighting*
Paper 1	2 hours	Short answer questions testing more on the fundamental skills and concepts	80	50%
Paper 2	$2\frac{1}{2}$ hours	Questions testing more on the higher order thinking skills	100	50%

NOTES

1. Paper 1 will consist of about 25 short answer questions. Candidates are required to answer all the questions. Paper 2 will consist of 2 sections. Section A will contain 9 to 10 questions with no choice. Section B will contain 2 questions of which candidates will be required to answer only one. Each choice carries the same number of marks, that is, between 10 to 12 marks.
2. Omission of essential working will result in loss of marks.
3. Spaces will be provided on the question paper of Paper 1 for working and answers.
4. Candidates are expected to cover the whole syllabus. Each paper may contain questions on any part of the syllabus and questions will not necessarily be restricted to a single topic.
5. Scientific calculators are allowed in Paper 2 but not in Paper 1.
6. Candidates should also have geometrical instruments with them for Paper 1 and Paper 2.
7. Paper 2 will be scheduled on a different day after Paper 1.
8. Unless stated otherwise within an individual question, three-figure accuracy will be required for answers in Paper 2. This means that four-figure accuracy should be shown throughout the working, including cases where answers are used in subsequent parts of the question. Premature approximation will be penalised, where appropriate.
9. SI units will be used in questions involving mass and measures: the use of the centimetre will continue.

   Both the 12-hour and 24-hour clock may be used for quoting times of the day. In the 24-hour clock, for example, 3.15 a.m. will be denoted by 03 15; 3.15 p.m. by 15 15, noon by 12 00 and midnight by 24 00.

   Candidates will be expected to be familiar with the solidus notation for the expression of compound units, e.g. 5cm/s for 5 centimetres per second, 13.6g/cm^3 for 13.6 grams per cubic centimetre.
10. Unless the question requires the answer in terms of π, use $\pi = 3.14$ for Paper 1, and use either your calculator value for π or $\pi = 3.142$ for Paper 2.

4017 MATHEMATICS O LEVEL

NO	TOPIC	SUBJECT CONTENT
1	Numbers	• use natural numbers, integers (positive, negative and zero), prime numbers, common factors and common multiples, rational and irrational numbers, real numbers; • continue given number sequences, recognise patterns within and across different sequences and generalise to simple algebraic statements (including expressions for the *n*th term) relating to such sequences.
2	Squares, square roots, cubes and cube roots	• calculate squares, square roots, cubes and cube roots of numbers.
3	Vulgar and decimal fractions and percentages	• use the language and notation of simple vulgar and decimal fractions and percentages in appropriate contexts; • recognise equivalence and convert between these forms.
4	Ordering	• order quantities by magnitude and demonstrate familiarity with the symbols $=, \neq, >, <, \geq, \leq$.
5	Standard form	• use the standard form $A \times 10^n$ where n is a positive or negative integer, and $1 \leq A < 10$.
6	The four operations	• use the four operations for calculations with whole numbers, decimal fractions and vulgar (and mixed) fractions, including correct ordering of operations and use of brackets.
7	Estimation	• make estimates of numbers, quantities and lengths; • give approximations to specified numbers of significant figures and decimal places; • round off answers to reasonable accuracy in the context of a given problem.
8	Ratio, proportion, rate	• demonstrate an understanding of the elementary ideas and notation of ratio, direct and inverse proportion and common measures of rate; • divide a quantity in a given ratio; • use scales in practical situations; • calculate average speed; • express direct and inverse variation in algebraic terms and use this form of expression to find unknown quantities.
9	Percentages	• calculate a given percentage of a quantity; • express one quantity as a percentage of another; • calculate percentage increase or decrease; • carry out calculations involving reverse percentages, e.g. finding the cost price given the selling price and the percentage profit.
10	Use of a scientific calculator	• use a scientific calculator efficiently; • apply appropriate checks of accuracy.

11	Everyday mathematics	• use directed numbers in practical situations (e.g. temperature change, tide levels); • use current units of mass, length, area, volume, capacity and time in practical situations (including expressing quantities in terms of larger or smaller units); • calculate times in terms of the 12-hour and 24-hour clock (including reading of clocks, dials and timetables); • solve problems involving money and convert from one currency to another; • use given data to solve problems on personal and household finance involving earnings, simple interest, compound interest (without the use of formula), discount, profit and loss; • extract data from tables and charts.
12	Graphs in practical situations	• interpret and use graphs in practical situations including travel graphs and conversion graphs; • draw graphs from given data; • apply the idea of rate of change to easy kinematics involving distance-time and speed-time graphs, acceleration and retardation; • calculate distance travelled as area under a linear speed-time graph.
13	Graphs of functions	• construct tables of values and draw graphs for functions of the form $y = ax^n$ where n = –2, –1, 0, 1, 2, 3, and simple sums of not more than three of these and for functions of the form $y = ka^x$ where a is a positive integer; • interpret graphs of linear, quadratic, reciprocal and exponential functions; • find the gradient of a straight line graph; • solve equations approximately by graphical methods; • estimate gradients of curves by drawing tangents.
14	Coordinate geometry	• demonstrate familiarity with cartesian coordinates in two dimensions; • calculate the gradient of a straight line from the coordinates of two points on it; • interpret and obtain the equation of a straight line graph in the form $y = mx + c$; • calculate the length and the coordinates of the midpoint of a line segment from the coordinates of its end points.
15	Algebraic representation and formulae	• use letters to express generalised numbers and express basic arithmetic processes algebraically; • substitute numbers for words and letters in formulae; • transform simple and more complicated formulae; • construct equations from given situations.
16	Algebraic manipulation	• manipulate directed numbers; • use brackets and extract common factors; • expand products of algebraic expressions; • factorise expressions of the form $ax + ay$; $ax + bx + kay + kby$; $a^2x^2 - b^2y^2$; $a^2 + 2ab + b^2$; $ax^2 + bx + c$; • manipulate simple algebraic fractions

4017 MATHEMATICS O LEVEL

17	Indices	• use and interpret positive, negative, zero and fractional indices.
18	Solutions of equations and inequalities	• solve simple linear equations in one unknown; • solve fractional equations with numerical and linear algebraic denominators; • solve simultaneous linear equations in two unknowns; • solve quadratic equations by factorisation and either by use of the formula or by completing the square; • solve simple linear inequalities.
19	Geometrical terms and relationships	• use and interpret the geometrical terms: point, line, plane, parallel, perpendicular, right angle, acute, obtuse and reflex angles, interior and exterior angles, regular and irregular polygons, pentagons, hexagons, octagons, decagons; • use and interpret vocabulary of triangles, circles, special quadrilaterals; • solve problems (including problems leading to some notion of proof) involving similarity and congruence; • use and interpret vocabulary of simple solid figures: cube, cuboid, prism, cylinder, pyramid, cone, sphere; • use the relationships between areas of similar triangles, with corresponding results for similar figures and extension to volumes of similar solids.
20	Geometrical constructions	• measure lines and angles; • construct simple geometrical figures from given data using protractors or set squares as necessary; • construct angle bisectors and perpendicular bisectors using straight edges and compasses only; • read and make scale drawings. (Where it is necessary to construct a triangle given the three sides, ruler and compasses only must be used.)
21	Bearings	• interpret and use three-figure bearings measured clockwise from the north (i.e. 000°–360°).
22	Symmetry	• recognise line and rotational symmetry (including order of rotational symmetry) in two dimensions, and properties of triangles, quadrilaterals and circles directly related to their symmetries; • recognise symmetry properties of the prism (including cylinder) and the pyramid (including cone); • use the following symmetry properties of circles: (a) equal chords are equidistant from the centre; (b) the perpendicular bisector of a chord passes through the centre; (c) tangents from an external point are equal in length.
23	Angle	• calculate unknown angles and solve problems (including problems leading to some notion of proof) using the following geometrical properties: (a) angles on a straight line; (b) angles at a point; (c) vertically opposite angles; (d) angles formed by parallel lines;

(e) angle properties of triangles and quadrilaterals;
(f) angle properties of polygons including angle sum;
(g) angle in a semi-circle;
(h) angle between tangent and radius of a circle;
(i) angle at the centre of a circle is twice the angle at the circumference;
(j) angles in the same segment are equal;
(k) angles in opposite segments are supplementary.

24 Locus

- use the following loci and the method of intersecting loci:
  (a) set of points in two dimensions
    (i) which are at a given distance from a given point,
    (ii) which are at a given distance from a given straight line,
    (iii) which are equidistant from two given points;
  (b) sets of points in two dimensions which are equidistant from two given intersecting straight lines.

25 Mensuration

- solve problems involving
  (i) the perimeter and area of a rectangle and a triangle,
  (ii) the circumference and area of a circle,
  (iii) the area of a parallelogram and a trapezium,
  (iv) the surface area and volume of a cuboid, cylinder, prism, sphere, pyramid and cone. (Formulae will be given for the sphere, pyramid and cone.)
  (v) arc length and sector area as fractions of the circumference and area of a circle.

26 Trigonometry

- apply Pythagoras theorem and the sine, cosine and tangent ratios for acute angles to the calculation of a side or of an angle of a right-angled triangle (angles will be quoted in, and answers required in, degrees and decimals of a degree to one decimal place);
- solve trigonometrical problems in two dimensions including those involving angles of elevation and depression and bearings;
- extend sine and cosine functions to angles between 90° and 180°;
- solve problems using the sine and cosine rules for any triangle and the formula $\frac{1}{2}ab\sin C$ for the area of a triangle;
- solve simple trigonometrical problems in three dimensions. (Calculations of the angle between two planes or of the angle between a straight line and a plane will not be required.)

27 Statistics

- collect, classify and tabulate statistical data;
- read, interpret and draw simple inferences from tables and statistical diagrams;
- construct and use bar charts, pie charts, pictograms, dot diagrams, stem-and-leaf diagrams, simple frequency distributions and frequency polygons;
- use frequency density to construct and read histograms with equal and unequal intervals;
- calculate the mean, median and mode for individual data and distinguish between the purposes for which they are used;
- construct and use cumulative frequency diagrams;

		• estimate the median, percentiles, quartiles and interquartile range from the cumulative frequency diagrams; • calculate the mean for grouped data; • identify the modal class from a grouped frequency distribution.
28	Probability	• calculate the probability of a single event as either a fraction or a decimal (not a ratio); • calculate the probability of simple combined events, using possibility diagrams and tree diagrams where appropriate (in possibility diagrams outcomes will be represented by points on a grid and in tree diagrams outcomes will be written at the end of branches and probabilities by the side of the branches).
29	Transformations	• use the following transformations of the plane: reflection (M), rotation (R), translation (T), enlargement (E), shear (H), stretch (S) and their combinations (if M(a)=b and R(b)=c the notation RM(a)=c will be used; invariants under these transformations may be assumed); • identify and give precise descriptions of transformations connecting given figures;
30	Vectors in two dimensions	• describe a translation by using a vector represented by $\begin{pmatrix} x \\ y \end{pmatrix}$, $\overrightarrow{AB}$ or $\mathbf{a}$; • add vectors and multiply a vector by a scalar; • calculate the magnitude of a vector $\begin{pmatrix} x \\ y \end{pmatrix}$, as $\sqrt{(x^2+y^2)}$. (Vectors will be printed as $\overrightarrow{AB}$ or $\mathbf{a}$ and their magnitudes denoted by modulus signs, e.g. $\lvert\overrightarrow{AB}\rvert$ or $\lvert\mathbf{a}\rvert$. In their answers to questions candidates are expected to indicate $\mathbf{a}$ in some definite way, e.g. by an arrow or by underlining, thus $\overrightarrow{AB}$ or $\underline{a}$); • represent vectors by directed line segments; • use the sum and difference of two vectors to express given vectors in terms of two coplanar vectors; • use position vectors.

MATHEMATICAL NOTATION

The list which follows summarizes the notation used in the Syndicate's Mathematics examinations. Although primarily directed towards Advanced level, the list also applies, where relevant, to examinations at all other levels, i.e. O level, AO level and N level.

Mathematical Notation

1. *Set Notation*

$\in$	is an element of
$\notin$	is not an element of
$\{x_1, x_2, \ldots\}$	the set with elements $x_1, x_2, \ldots$
$\{x: \ldots\}$	the set of all x such that …
$\mathrm{n}(A)$	the number of elements in set A
$\varnothing$	the empty set
$\mathscr{E}$	universal set
A'	the complement of the set A
$\mathbb{N}$	the set of positive integers, $\{1, 2, 3, \ldots\}$
$\mathbb{Z}$	the set of integers $\{0, \pm1, \pm2, \pm3, \ldots\}$
$\mathbb{Z}^+$	the set of positive integers $\{1, 2, 3, \ldots\}$
$\mathbb{Z}_n$	the set of integers modulo n, $\{0, 1, 2, \ldots, n-1\}$
$\mathbb{Q}$	the set of rational numbers
$\mathbb{Q}^+$	the set of positive rational numbers, $\{x \in \mathbb{Q}: x > 0\}$
$\mathbb{Q}_0^+$	the set of positive rational numbers and zero, $\{x \in \mathbb{Q}: x \geqslant 0\}$
$\mathbb{R}$	the set of real numbers
$\mathbb{R}^+$	the set of positive real numbers $\{x \in \mathbb{R}: x > 0\}$
$\mathbb{R}_0^+$	the set of positive real numbers and zero $\{x \in \mathbb{R}: x \geqslant 0\}$
$\mathbb{R}^n$	the real n tuples
$\mathbb{C}$	the set of complex numbers
$\subseteq$	is a subset of
$\subset$	is a proper subset of
$\nsubseteq$	is not a subset of
$\not\subset$	is not a proper subset of
$\cup$	union
$\cap$	intersection
$[a, b]$	the closed interval $\{x \in \mathbb{R}: a \leqslant x \leqslant b\}$
$[a, b)$	the interval $\{x \in \mathbb{R}: a \leqslant x < b\}$
$(a, b]$	the interval $\{x \in \mathbb{R}: a < x \leqslant b\}$
(a, b)	the open interval $\{x \in \mathbb{R}: a < x < b\}$
yRx	y is related to x by the relation R
$y \sim x$	y is equivalent to x, in the context of some equivalence relation

2. *Miscellaneous Symbols*

$=$	is equal to
$\neq$	is not equal to
$\equiv$	is identical to or is congruent to
$\approx$	is approximately equal to
$\cong$	is isomorphic to
$\propto$	is proportional to
$<$; $\ll$	is less than; is much less than
$\leqslant$, $\ngtr$	is less than or equal to or is not greater than
$>$; $\gg$	is greater than; is much greater than
$\geqslant$, $\nless$	is greater than or equal to or is not less than
∞	infinity

3. *Operations*

$a + b$	a plus b
$a - b$	a minus b
$a \times b$, ab, $a.b$	a multiplied by b
$a \div b$, $\frac{a}{b}$, a/b	a divided by b
$a : b$	the ratio of a to b
$\sum_{i=1}^{n} a_i$	$a_1 + a_2 + \ldots + a_n$
$\sqrt{a}$	the positive square root of the real number a
$\lvert a \rvert$	the modulus of the real number a
$n!$	n factorial for $n \in \mathbb{N}$ $(0! = 1)$
$\binom{n}{r}$	the binomial coefficient $\frac{n!}{r!\,(n-r)!}$, for $n, r \in \mathbb{N}$, $0 \leqslant r \leqslant n$ $\frac{n(n-1)\ldots(n-r+1)}{r!}$, for $n \in \mathbb{Q}$, $r \in \mathbb{N}$

4. *Functions*

f	function f
f(x)	the value of the function f at x
f : $A \rightarrow B$	f is a function under which each element of set A has an image in set B
f : $x \mapsto y$	the function f maps the element x to the element y
f^{-1}	the inverse of the function f
$\mathrm{g} \circ \mathrm{f}$, gf	the composite function of f and g which is defined by $(\mathrm{g} \circ \mathrm{f})(x)$ or $\mathrm{gf}(x) = \mathrm{g}(\mathrm{f}(x))$
$\lim_{x \to a} \mathrm{f}(x)$	the limit of f(x) as x tends to a
Δx; δx	an increment of x
$\frac{\mathrm{d}y}{\mathrm{d}x}$	the derivative of y with respect to x
$\frac{\mathrm{d}^n y}{\mathrm{d}x^n}$	the nth derivative of y with respect to x
$\mathrm{f}'(x), \mathrm{f}''(x), \ldots, \mathrm{f}^{(n)}(x)$	the first, second, . . . , nth derivatives of f(x) with respect to x
$\int y\,\mathrm{d}x$	indefinite integral of y with respect to x
$\int_a^b y\,\mathrm{d}x$	the definite integral of y with respect to x for values of x between a and b
$\frac{\partial y}{\partial x}$	the partial derivative of y with respect to x
$\dot{x}, \ddot{x}, \ldots$	the first, second, . . . derivatives of x with respect to time.

5. *Exponential and Logarithmic Functions*

e	base of natural logarithms
e^x, $\exp x$	exponential function of x
$\log_a x$	logarithm to the base a of x
$\ln x$	natural logarithm of x
$\lg x$	logarithm of x to base 10

MATHEMATICAL NOTATION

6. *Circular and Hyperbolic Functions and Relations*

sin, cos, tan, cosec, sec, cot	the circular functions
$\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$, $\operatorname{cosec}^{-1}$, $\sec^{-1}$, $\cot^{-1}$	the inverse circular relations
sinh, cosh, tanh, cosech, sech, coth	the hyperbolic functions
$\sinh^{-1}$, $\cosh^{-1}$, $\tanh^{-1}$, $\operatorname{cosech}^{-1}$, sech^{-1}, $\coth^{-1}$	the inverse hyperbolic relations

7. *Complex Numbers*

i	square root of –1
z	a complex number, $z = x + \mathrm{i}y$ $= r(\cos\theta + \mathrm{i}\sin\theta)$, $r \in \mathbb{R}_0^+$ $= r\mathrm{e}^{\mathrm{i}\theta}$, $r \in \mathbb{R}_0^+$
Re z	the real part of z, $\mathrm{Re}(x + \mathrm{i}y) = x$
Im z	the imaginary part of z, $\mathrm{Im}(x + \mathrm{i}y) = y$
$\lvert z \rvert$	the modulus of z, $\lvert x + \mathrm{i}y \rvert = \sqrt{(x^2 + y^2)}$, $\lvert r(\cos\theta + \mathrm{i}\sin\theta) \rvert = r$
arg z	the argument of z, $\arg(r(\cos\theta + \mathrm{i}\sin\theta)) = \theta$, $-\pi < \theta \leqslant \pi$
z^*	the complex conjugate of z, $(x + \mathrm{i}y)^* = x - \mathrm{i}y$

8. *Matrices*

$\mathbf{M}$	a matrix $\mathbf{M}$
$\mathbf{M}^{-1}$	the inverse of the square matrix $\mathbf{M}$
$\mathbf{M}^{\mathrm{T}}$	the transpose of the matrix $\mathbf{M}$
det $\mathbf{M}$	the determinant of the square matrix $\mathbf{M}$

9. *Vectors*

$\mathbf{a}$	the vector $\mathbf{a}$
$\overrightarrow{AB}$	the vector represented in magnitude and direction by the directed line segment AB
$\hat{\mathbf{a}}$	a unit vector in the direction of the vector $\mathbf{a}$
$\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$	unit vectors in the directions of the cartesian coordinate axes
$\lvert \mathbf{a} \rvert$	the magnitude of $\mathbf{a}$
$\lvert \overrightarrow{AB} \rvert$	the magnitude of $\overrightarrow{AB}$
$\mathbf{a} \cdot \mathbf{b}$	the scalar product of $\mathbf{a}$ and $\mathbf{b}$
$\mathbf{a} \times \mathbf{b}$	the vector product of $\mathbf{a}$ and $\mathbf{b}$

10. *Probability and Statistics*

A, B, C, etc.	events
$A \cup B$	union of events A and B
$A \cap B$	intersection of the events A and B
P(A)	probability of the event A
A'	complement of the event A, the event ‘not A’
P(A|B)	probability of the event A given the event B
X, Y, R, etc.	random variables
x, y, r, etc.	values of the random variables X, Y, R, etc.
$x_1, x_2, \ldots$	observations
$f_1, f_2, \ldots$	frequencies with which the observations $x_1, x_2, \ldots$ occur
p(x)	the value of the probability function $\mathrm{P}(X = x)$ of the discrete random variable X

MATHEMATICAL NOTATION

$p_1, p_2, \ldots$	probabilities of the values $x_1, x_2, \ldots$ of the discrete random variable X.
$\mathrm{f}(x), \mathrm{g}(x), \ldots$	the value of the probability density function of the continuous random variable X
$\mathrm{F}(x), \mathrm{G}(x), \ldots$	the value of the (cumulative) distribution function $\mathrm{P}(X \leqslant x)$ of the random variable X
$\mathrm{E}(X)$	expectation of the random variable X
$\mathrm{E}[\mathrm{g}(X)]$	expectation of $\mathrm{g}(X)$
$\mathrm{Var}(X)$	variance of the random variable X
$\mathrm{G}(t)$	the value of the probability generating function for a random variable which takes integer values
$\mathrm{B}(n, p)$	binomial distribution, parameters n and p
$\mathrm{N}(\mu, \sigma^2)$	normal distribution, mean μ and variance σ^2
μ	population mean
σ^2	population variance
σ	population standard deviation
$\bar{x}$	sample mean
s^2	unbiased estimate of population variance from a sample, $s^2 = \frac{1}{n-1}\sum(x-\bar{x})^2$
ϕ	probability density function of the standardised normal variable with distribution N(0, 1)
Φ	corresponding cumulative distribution function
ρ	linear product-moment correlation coefficient for a population
r	linear product-moment correlation coefficient for a sample
$\mathrm{Cov}(X, Y)$	covariance of X and Y